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Gravitino warm dark matter with entropy production

Edward A. Baltz
Columbia Astrophysics Laboratory
550 W 120th St., Mail Code 5247, New York, NY 10027, USA
E-mail: eabaltz@physics.columbia.edu

Hitoshi Murayama
Theory Group, Lawrence Berkeley National Laboratory and Department of Physics
University of California, Berkeley, CA 94720, USA
E-mail: murayama@lbl.gov

Abstract: Gravitinos with a mass in the keV range are an interesting candidate for warm dark matter. Recent measurements of the matter density of the universe and of cosmic structures at the dwarf galaxy scale rule out the simplest gravitino models with thermal freeze-out. We construct a model where the decay of the messenger particles that transmit the supersymmetry breaking to the observable sector generates the required entropy to dilute the gravitino relic density by the required factor of a few to come in line with observations. The model is natural, and requires only that the coupling of the messenger sector to the standard model be set so that the decay happens at the appropriate time.

Keywords: Supersymmetry Phenomenology, Dark Matter.

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Contents

1. Introduction

We revisit the possibility that gravitinos could be the dominant component of nonrelativistic matter in the universe. Especially interesting to us are scenarios where the gravitino is very light, with a mass in the keV range. Such a species would be "warm" dark matter.

This possibility is interesting for two reasons, which we will now enumerate. The first is that such a light gravitino arises naturally in models of gauge-mediated supersymmetry breaking [1] (GMSB) revived recently [2]. They are stable, and thus could potentially pose problems for cosmology [4, 5, 6], which we will address in light of new more accurate measurements of both the matter density of the universe and on constraints on deviations from cold dark matter.

Our second motivation is that recently cold dark matter has come under fire, and warm dark matter may in fact be favored. In particular, the predictions of the cold dark matter theory indicate that galaxies such as the Milky Way should have a large number of satellites, which are not observed. Warm dark matter could ameliorate this problem, as density fluctuations at the satellite scale and smaller are suppressed [8, 9]. In these models, the densities of halo cores are lowered, large halos have many fewer subhalos, and the subhalos that are present form in a top-down fashion, namely by fragmentation of larger clumps. This is contrary to the prediction of cold dark matter, where all relevant structures form in a bottom-up fashion by mergers of smaller structures [9].

As data have improved, both in terms of the matter density of the universe, and in constraints on deviations from collisionless dark matter, the gravitino has silently stopped being a viable dark matter candidate as its relic density is too large in the simple thermal freeze-out scenario. We rescue the gravitino in GMSB models by invoking a natural model for entropy production, and thus provide again a candidate for warm dark matter, with a mass in the range $1.0 - 1.5$ keV.

JHEP05(2003)067

2. Gravitino mass and relic density

In GMSB models, the gravitino mass $m_{3/2}$ is related to the SUSY breaking scale $\Lambda_{\rm SUSY}$ as follows,

$$m_{3/2} = \frac{1}{\sqrt{3}} \left(\frac{\Lambda_{\rm SUSY}^2}{M_{\rm pl}} \right) = 0.237 \ \text{keV} \ \left(\frac{\Lambda_{\rm SUSY}}{\rm PeV} \right)^2 , \tag{2.1}$$

where $M_{\rm pl}$ is the reduced Planck mass. We will be concerned with gravitino masses around 1 keV, thus SUSY breaking scales of 2 PeV, which are natural in GMSB models.[1]

Gravitinos in this scenario interact fairly strongly, and thus freeze out quite late, at a temperature of order the weak scale or even colder. As they are of course highly relativistic, the computation of their relic density is relatively simple: all that needs to be done is to compute the effective degrees of freedom g_* at freeze-out. Their relic density is then obtained,

$$\Omega_{3/2} h^2 = 1.14 \left(\frac{g_*}{100} \right)^{-1} \left(\frac{m_{3/2}}{\rm keV} \right) . \tag{2.2}$$

The value of g_* at gravitino freeze-out has been determined to be in the range 90-140 for a wide range of parameters [6].

Finally we consider astrophysical constraints on the mass of the relic particle making up the dark matter. Particles in the eV range that undergo thermal freeze-out are ruled out as “hot” dark matter. The transition to “cold” dark matter occurs for thermal relics in the keV range. In fact, there is a lower bound on the mass of a warm relic [9, 10],

$$m_{3/2} > 0.75 - 1.0 \, \text{keV} \longrightarrow \Omega_{3/2} h^2 > 0.6 \, , \tag{2.3}$$

based on the requirement that the small scale fluctuations are not in conflict with e.g. the Lyα forest. This mass bound is valid for any *thermal* relic, but is modified for relics whose distribution functions deviate from equilibrium [11].

We compare this bound on the relic density of gravitinos with current data on the density of the dark matter, which favors $\Omega_{\rm DM} h^2 \approx 0.11$ [12], and in any circumstances $\Omega_{\rm DM} h^2 < 0.3$, indicating a serious discrepancy. One way to alleviate this is to allow for entropy production after gravitino freeze-out, which dilutes the gravitino relic density to acceptable levels. We discuss this possibility in the next section.

[1] In this class of models, so-called messenger particles acquire a mass M and a supersymmetry-breaking scalar mass term F. The superparticles in the SUSY Standard Model acquire soft masses via gauge interactions at the 2-loop level $m^2_{\rm soft} = \sum_a 2C_a \left(\frac{g_a^2}{16\pi^2}\right)^2 \left(\frac{F}{M}\right)^2$, where g_a are gauge coupling constants and C_a are quadratic Casimir. In the original model of gauge mediation, $\sqrt{F} \sim M$ is suppressed relative to the primordial supersymmetry breaking scale $\Lambda_{\rm SUSY}$ by a loop-factor, and the gravitino mass is typically heavier than 50 keV [5]. But in models of direct gauge mediation [3], $\sqrt{F} \sim \Lambda_{\rm SUSY}$ with no loop suppression. In particular, the model in ref. [7] achieves $m_{3/2} \sim 1$ keV naturally. We thus can have standard weak-scale supersymmetry in the rest of the superparticle spectrum, with the caveat being that the gravitino is the LSP and all other superparticles decay to gravitinos and Standard Model particles.

3. Entropy production: simple model

The production of entropy is required to dilute the relic density of gravitinos. As current data favor $\Omega_{\rm DM}h^2 \approx 0.11$, and in any event $\Omega_{\rm DM}h^2 < 0.3$, we will consider a comoving entropy density increase of a factor of several.

In GMSB models there is a messenger sector around the SUSY breaking scale, which for keV gravitinos is of order 2 PeV. As our entropy production mechanism we will consider the out-of-equilibrium decays of the messengers via small couplings to the Standard Model. Assuming stability of the messengers, their relic density has been calculated [13], and we simply quote the results here,

$$\Omega_M h^2 \approx 10^5 \left(\frac{M}{\rm PeV}\right)^2 \longrightarrow Y_M = \frac{\Omega_M \rho_c}{s_0 M} \approx 3.65 \times 10^{-10} \left(\frac{M}{\rm PeV}\right), \tag{3.1}$$

where M is the mass of the messenger particle, Y_M is the frozen-out value of n/s, the ratio of number and entropy densities for the messenger particles, s_0 is the entropy density today, and ρ_c is the critical density today.

In this section we will assume that the messengers decay instantaneously when the temperature of the thermal bath is T_D, and their Standard Model decay products thermalize instantaneously also. This approximation requires only the conservation of energy. Before decay, the energy density is

$$\rho = \frac{\pi^2}{30} g_* T^4 + \frac{2\pi^2}{45} g_* T^3 M Y_M \,. \tag{3.2}$$

The first term is the radiation density and the second is the matter density of messengers. This quantity is simply equated to the radiation density afterwards, with a higher temperature T', and no matter, giving the temperature increase. This is then simply related to the fractional increase in entropy density s'/s.

$$\left(\frac{T'}{T}\right)^4 = \left(\frac{s'}{s}\right)^{4/3} = 1 + \frac{4}{3} Y_M \left(\frac{M}{T}\right) = 1 + 4.87 \left(\frac{M}{10~{\rm PeV}}\right)^2 \left(\frac{T_D}{10~{\rm MeV}}\right)^{-1}. \tag{3.3}$$

Here we gave the temperature in MeV to be suggestive. The temperature at decay must be larger than this so as not to disrupt the nucleosynthesis era. To be safe, $T_D > 10$ MeV is warranted.

To be specific, consider a gravitino with mass 1 keV, and thus $\Omega_{3/2}h^2 \approx 1$. Assuming we want $\Omega_{\rm DM}h^2 = 0.15$, we must dilute by a factor of $s'/s = 6$. For this model we find that the temperature increases by a factor of $T'/T \approx 1.8$ during the decay. We now find a simple relation between the mass of the messenger and the temperature at which it decays,

$$\left(\frac{M}{10~{\rm PeV}}\right)^2 \approx 2 \left(\frac{T_D}{10~{\rm MeV}}\right). \tag{3.4}$$

All that remains to be done in this model is to adjust the coupling between the messengers and the Standard Model particles to enforce this constraint on the lifetime τ of the messengers,

$$\tau \equiv \frac{1}{\Gamma} = \frac{1}{2H} \equiv \frac{M_{\rm pl}}{T_D^2}\sqrt{\frac{45}{2\pi^2 g_*}} \longrightarrow \left(\frac{T_D}{10~{\rm MeV}}\right)^2 \approx 1.11 \times 10^{29}\, \frac{\Gamma}{10\,{\rm PeV}}\,. \tag{3.5}$$

Note that here g_* is the effective number of degrees of freedom at the decay, which for temperatures between 1 and 100 MeV is $g_* = 43/4$. Therefore, the coupling of messenger particles to the standard model particles must be extremely suppressed. This is actually a desirable feature so as to not spoil the flavor-independence of the gauge-mediated supersymmetry breaking [14].

We now choose the form of the interactions between the messengers and the Standard Model particles. Such interactions of course violate messenger number, allowing the lightest messenger to decay. The simplest possibility is to include messenger number violating cubic terms in the superpotential, allowing two-body decays [15]. These interactions are similiar to the usual R-parity violating terms that can be added to the MSSM. In this case, the decay rate $\Gamma \sim g^2 M$, which would require the coupling g^2 to be extremely small for the messengers to live long enough to suit our purposes. Another possibility is a higher dimension operator, or equivalently, an exchange of a heavy particle. For example, it is possible to construct models where the Standard Model particles and the messengers interact directly only through spontaneously broken gauge interactions. One way to do so is to introduce a non-abelian horizontal gauge symmetry among Standard Model particles (say, $\mathbf{5}^*$ in SU(5) language) and messengers with the same quantum numbers ($\mathbf{5}^*$). Alternatively, one can introduce a superpotential coupling of a heavy chiral multiplet to messenger field and a standard model field while its conjugate field couples to light fields only. In either case, the decay is mediated by heavy particles (e.g, gauge bosons) of mass M_X, allowing a three-body decay. The lifetime is

$$\Gamma \approx \frac{g^4}{192\pi^3}\frac{M^5}{M_X^4} \to \left(\frac{T_D}{10\ \mathrm{MeV}}\right)^2 \approx 18.6\left(\frac{g^2}{0.01}\right)^2\left(\frac{M}{10\ \mathrm{PeV}}\right)^5\left(\frac{M_X}{10^{12}\ \mathrm{GeV}}\right)^{-4}. \tag{3.6}$$

Thus, a heavy particle of mass of 10^{12} GeV coupling the messenger sector to the standard model gives roughly the right lifetime. This is somewhat lower than the GUT scale, but still far above the messenger scale, so these interactions should not invalidate the relic density prediction.

4. Entropy production: full evolution

In the previous section we assumed that the messengers decay instantaneously. This approach allows us to study entropy production without lengthy calculations. The results from a more accurate calculation are similar, as we will show [16]. In reality of course, the decay is exponential, and we should follow the evolution of matter and radiation through the decay process to accurately determine the increase in entropy, and the relative change in gravitino temperature. In this section we will do just that. We find a similar increase in entropy as for the simple calculation, though we also show that the temperature never increases, it merely decreases more slowly.

We now write down the equations for the evolution of the radiation and matter densities, and the expansion scale factor. We assume that the universe is flat, with no cosmological constant, as is appropriate for early times. The equations are [16]

$$\dot{\rho}_M + 3H\rho_M = -\Gamma\rho_M\,, \tag{4.1}$$

$$S^{1/3}\dot{S} = \left(\frac{2\pi^2}{45}\,g_*\right)^{1/3} \Gamma\rho_M R^4, \tag{4.2}$$

$$H^2 \equiv \left(\frac{1}{R}\frac{dR}{dt}\right)^2 = \frac{1}{3M_{\rm pl}^2}\left(\rho_M + \rho_R\right), \tag{4.3}$$

where $S = sR^3$ is the comoving entropy density. The first equation is solved trivially,

$$\rho_M(t) = \rho_M(t_i)\left(\frac{R(t)}{R(t_i)}\right)^{-3} e^{-\Gamma(t-t_i)}, \tag{4.4}$$

and we solve the remaining two equations numerically. We recover the entropy increase from the solution,

$$\frac{s'}{s} = \frac{S(t_f)}{S(t_i)}. \tag{4.5}$$

The effective temperature decrease of the gravitino is also easily recovered as

$$\frac{T_R(t_f)/T_R(t_i)}{T_{3/2}(t_f)/T_{3/2}(t_i)} = \left(\frac{s'}{s}\right)^{1/3}, \tag{4.6}$$

in other words the radiation temperature decreases less than the gravitino temperature. This indicates that for a given mass, the gravitino behaves more like cold dark matter than expected.

Considering the results of the previous section, we choose two fiducial cases to run numerically. We use decay temperatures of 10 and 100 MeV, requiring $M \approx 14, 45$ PeV respectively, and we set the decay times as appropriate. Evolving eqs. (4.1)–(4.3) for these parameters, we arrive at figure 1. We start the evolution at a temperature of 10 GeV, and take $g_*(T)$ for entropy from ref. [17]. The onset of matter domination and the entropy-producing decay are both clear. By a temperature of a few MeV, the decay is all but complete, and nucleosynthesis can proceed as normal. We note that in principle, an epoch of matter domination such as is present in our model affects the evolution of the density fluctuations responsible for structure formation. In our case, the effects occur at wavelengths far too short to be observed in the cosmic microwave background or in large scale structure.

5. Discussion

We have constructed a model for gravitino warm dark matter that has the appropriate relic density. Entropy is produced by the decay of the messenger particles. The lifetime of the messengers is of the correct order if the decay happens through heavy particles of 10^{12} GeV or so.

For the keV gravitino we consider, the temperature, and thus the momenta, are decreased relative to the radiation by a factor of approximately two. This modifies the free-streaming scale $R_{\rm fs}$ of the gravitinos, and thus the effect on structure formation. This

JHEP05(2003)067

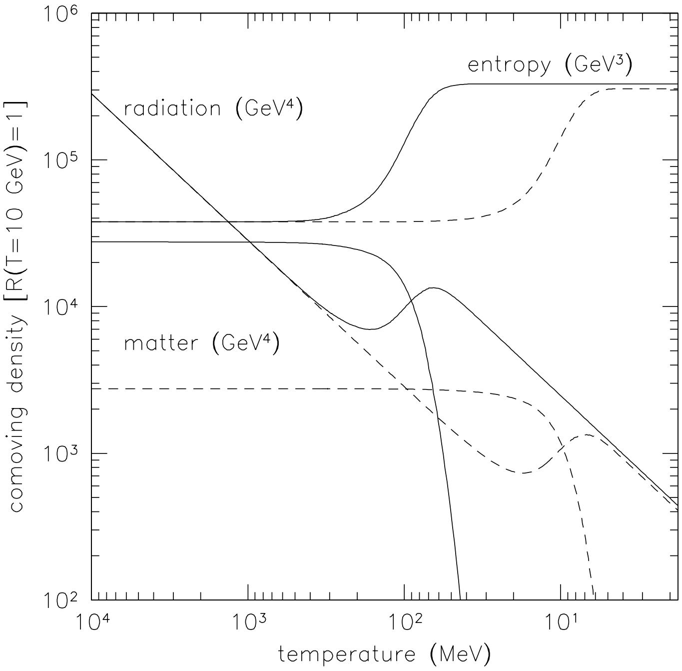

Figure 1: Evolution of entropy density. Here we show the decay of messengers at temperatures of 100 MeV (solid curve) and 10 MeV (dashed curve), producing an entropy increase of about a factor of eight. The onset of matter domination at a temperature of about 1 GeV (100 MeV) is clearly seen, and the decay at 100 MeV (10 MeV) is also clear.

length scale is proportional to the velocity of the particles at the epoch of matter-radiation equality, and thus $R_{\rm fs} \propto T_{3/2}/m_{3/2}$, normally given by [18, 19]

$$R_{\rm fs} \approx 0.2 \left(\frac{g_*}{100}\right)^{-1/3} \left(\frac{m_{3/2}}{\rm keV}\right)^{-1} {\rm Mpc} \approx 0.2 \left(\Omega_{3/2}h^2\right)^{1/3} \left(\frac{m_{3/2}}{\rm keV}\right)^{-4/3} {\rm Mpc}\,, \qquad (5.1)$$

where in the second relation the expression for the relic density is substituted for mass. In our case the relic density is different, and we must be careful in replacing the expression. With entropy production, the gravitinos are colder at matter-radiation equality than a similar constituent would be without entropy production. This free-streaming scale comes about at the epoch of matter-radiation equality, where the temperature is a few eV. The gravitinos at this time are non-relativistic, so we can simply decrease their velocity by the decrease in temperature. We thus find that roughly speaking,

$$R_{\rm fs} \approx 0.2 \left(\frac{g_*(s'/s)}{100}\right)^{-1/3} \left(\frac{m_{3/2}}{\rm keV}\right)^{-1} {\rm Mpc} \approx 0.2 \left(\Omega_{3/2}h^2\right)^{1/3} \left(\frac{m_{3/2}}{\rm keV}\right)^{-4/3} {\rm Mpc}\,. \qquad (5.2)$$

This expression is identical to the previous one. If we use the quantities $m_{3/2}$ and $\Omega_{3/2}h^2$ only, the expression for the free-streaming scale is unaffected by entropy production, at least in this approximation.

A more accurate approach accounts for free-streaming prior to matter-radiation equality [9, 11]. We simply quote the results here.

$$R_{\rm fs} \approx 0.65 \left(\Omega_m h^2\right)^{0.15} \left(\frac{m_{3/2}}{\rm keV}\right)^{-1.15} {\rm Mpc}\,. \qquad (5.3)$$

As the current bounds on the mass of a warm dark matter particle are in the $0.75 - 1.0$ keV range, we consider a gravitino slightly more massive than this, in the range $1.0 - 1.5$ keV. The beneficial qualities of the warm dark matter scenario for the most part persist for particles in this mass range [9]. The entropy production required is then

$$\frac{s'}{s} = 7.6 \left(\frac{g_*}{100}\right)^{-1} \left(\frac{\Omega_m h^2}{0.15}\right)^{-1} \frac{m_{3/2}}{\text{keV}}\,. \tag{5.4}$$

We have shown that entropy production of this order is not a serious difficulty, thus the gravitino is again a viable candidate for warm dark matter.

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